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VARIABLE PROPERTIES LAMINAR GAS FLOW HEAT TRANSFER IN THE ENTRY REGION OF PARALLEL POROUS PLATES

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Abstract—The principal effect of property variation is to slightly increase the Nusselt number in the thermal entry region. Previous investigations have given solutions to the flow and heat transfer problems over the entire range of wall Reynolds and Péclét numbers for a model of laminar, incompressible, steady, constant property flow. This note gives the designer an indication of the limitations of such a model. Specifically, for injection, eventually the Mach number and Reynolds number of the main channel flow will exceed the limits of the incompressible and laminar flow assumptions. For suction, the heat-transfer predictions cease to be valid near the point of complete mass extraction since the axial heat conduction now becomes important and cannot be assumed to be negligible.

NOMENCLATURE

- b, plate spacing;
- c_p, specific heat at constant pressure;
- D, hydraulic diameter, twice the plate spacing;
- H, enthalpy;
- k, thermal conductivity;
- Nu, Nusselt number, $q''_w D/\bar{k}(T_w - T_b)$;
- P, pressure;
- q''_w, $k(\partial T/\partial y)\,w$, wall heat flux;
- Q^+, dimensionless heat flux, $q''_w D/k_0 T_0$;
- Pe_w, wall Péclét number, $Re_w Pr$;
- Pr, Prandtl number;
- R, gas constant;
- Re, main channel Reynolds number, $\bar{U}D/\nu$;
- Re_w, wall Reynolds number, positive for injection, $V_w D/\nu$;
- T, fluid temperature;
- T^+, temperature ratio, T/T_0;
- T_b, local bulk fluid temperature;
- T_w, local wall temperature;
- u, axial velocity;
- $\bar{U}$, bulk axial velocity;
- v, transverse velocity;
- V_w, transverse velocity at the wall;
- x, axial Cartesian coordinate;
- X^+, inverse Graetz number, $x/DRe_0 Pr$;
- y, transverse Cartesian coordinate.

Greek letters
- η, dimensionless transverse coordinate, $1 - 2y/b$;
- μ, dynamic viscosity;
- ν, kinematic viscosity;
- ρ, density;
- $\bar{\rho}$, bulk density, P/RT_b.

Superscript
−, bulk conditions.

Subscripts
cp, constant property condition;
0, conditions at the thermal entrance;
w, conditions at the porous walls.

INTRODUCTION

THIS note provides results of an examination of the parallel porous plate entry problems for both constant wall temperature and constant wall heat flux boundary conditions for variable property, incompressible, steady, laminar gas flow with uniform injection or extraction. Constant property solutions to the thermal and combined entry problems have been reported in brief notes by the authors [1, 2]. Raithby [3] has also given solutions to the constant property thermal entry problem. The results of this note have application to process flow reactors where condensation or evaporation are present.

APPROACH

By neglecting viscous dissipation and axial heat conduction and by invoking the usual boundary layer assumptions, the local continuity, momentum and energy equations are:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\mathrm{d}P}{\mathrm{d}x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{2}$$

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right). \tag{3}$$

Note that the pressure is assumed to be dependent upon axial position only and is therefore constant across the channel at any axial location. The solution of variable properties flow is more difficult than the constant property case due to the coupling of the energy and momentum equations. Solutions are limited to ideal gases only. The coupling comes about through the variation of density and transport properties with temperature. This variation is treated by employing a power law dependency with temperature in the same manner as Worsøe-Schmidt and Leppert [4]

$$c_p/c_{p,0} = (T/T_0)^{0\cdot12} \tag{5}$$

$$\mu/\mu_0 = (T/T_0)^{0\cdot67} \tag{6}$$

$$k/k_0 = (T/T_0)^{0\cdot71} \tag{7}$$

where the exponents given are for air over the range 500–3000°R. The boundary conditions employed are no-slip and equal uniform injection (suction) of mass at the walls and constant, equal temperature walls or constant, equal heat flux through the walls. Holding the wall mass flux, $\rho_p V_w$, constant is probably the model closest to a real engineering application. The local bulk enthalpy is defined as

$$\bar{H} = \frac{\int_0^b \rho U H \,\mathrm{d}y}{\int_0^b \rho U \,\mathrm{d}y} \tag{8}$$

from which the bulk temperature, T_b, can be obtained. The injected or extracted fluid is assumed to be identical with that of the main channel flow and to be at local wall temperature when it enters or leaves the wall.

A relatively simple finite control volume analysis is used. The method, described in detail by Doughty [5], consists of a sequential solution of equations (1)–(3) at each axial station and is a marching integration routine which depends upon the values obtained at the previous station. Velocity and temperature predictions at each step are taken to be complete when their values have converged to the sixth decimal place. A 91 by 205 coordinate grid was used to accurately represent the flow development. A typical case requires about three minutes computation time on a CDC 6600 computer.

RESULTS

The local Nusselt number has been shown [1–3, 5] to be a function of the wall Reynolds

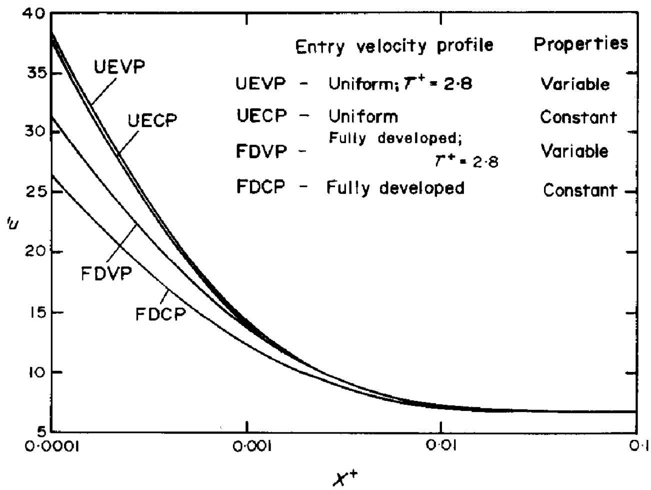

FIG. 1. Variation in heat transfer for several entry and property conditions, $Re_w = 2$, $Re_0 = 2083\frac{1}{3}$, constant wall temperature.

number and Prandtl number. When conditions at the wall are suddenly changed due to strong heating or cooling, both parameters are affected by property variation. By proper selection of T_w^+, one can obtain values for Re_w down-stream of the wall temperature change such that comparisons can be easily made between constant and variable property cases at the same Re_w.

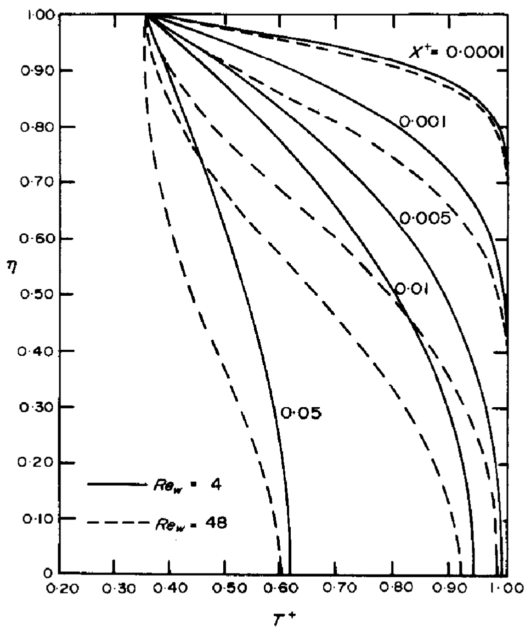

FIG. 2. Influence of injection upon the thermal development, cooling, $T_w^+ = 0{\cdot}355$.

The local Nusselt number is not noticeably affected by property variation. For the uniform velocity entry profile, the difference between the constant and variable property solutions for a step change in wall temperature is so small that the easier constant property solution can be used to represent the variable property also. The differences between the solutions are shown for a representative case in Fig. 1.

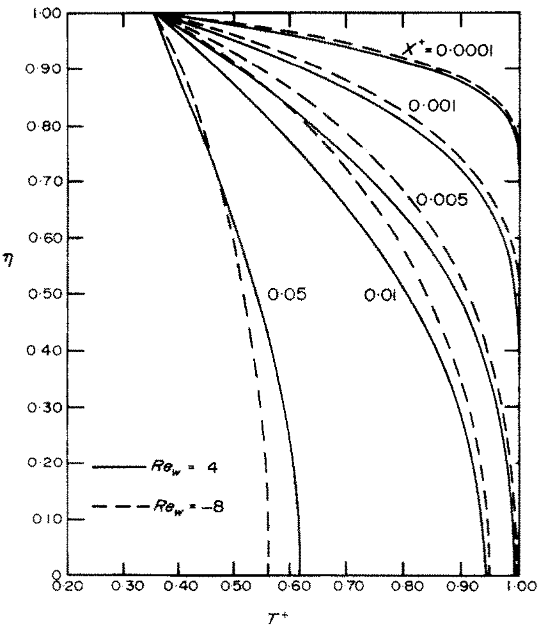

FIG. 3. Influence of suction upon the thermal development, cooling, $T_w^+ = 0{\cdot}355$.

Holding the wall temperature constant and varying the injection rate produces the expected result of increasing convection causing the temperature profile to develop somewhat more quickly in the major portion of the thermal entry region. This can be seen in Fig. 2. For suction cases, the temperature profile initially develops slower than a light injection case, but as the flow slows, the transverse conduction becomes the dominant mechanism and the profile develops rapidly as the point of complete mass extraction is approached as shown in Fig. 3.

A particularly important result is that for the case of heating and injection, the flow is strongly accelerated such that compressible flow must be considered for wall Péclét numbers over 30. Also if one considers designing a long channel ($X^+ > 0{\cdot}2$) of high aspect ratio where Pe_w exceeds 30, turbulent flow effects should also be included in the analysis as transition is expected.

The constant positive heat flux boundary condition results in a continuously increasing wall temperature as the flow proceeds down the channel. As a direct consequence, the density, viscosity, wall Reynolds number and Prandtl number also change continuously. With the wall mass flux maintained constant along the channel, the wall Reynolds number variation becomes

$$Re_w = \rho_w V_w D/\mu_w = \text{constant}/T_w^{0{\cdot}67} \qquad (9)$$

This indicates that with a continuously changing wall Reynolds number, fully developed solutions may not be possible, but instead one might have what could be called quasi-fully developed conditions for the flow parameters where the changes with axial displacement are small compared to those in the extreme entrance region ($X^+ < 0{\cdot}001$). Extending the problem to its limits, as the wall temperature increases to infinity, the wall Reynolds number goes to zero and thus one can expect the Nusselt number to behave in an asymptotic fashion towards 8·235. The approach to the asymptotic value is a function of the heating rate. This is especially evident for the suction case shown in Fig. 4 where the heat transfer with variable properties is compared to constant property cases. One actually sees two effects, the first is the expected increase in Nusselt number in the extreme entrance region due to the variable property effects similar to that found for the constant temperature heated wall case. The second effect is peculiar to the constant heat flux boundary condition. Because the wall Reynolds number changes continuously, the Nusselt number will increase for flow with injection through the

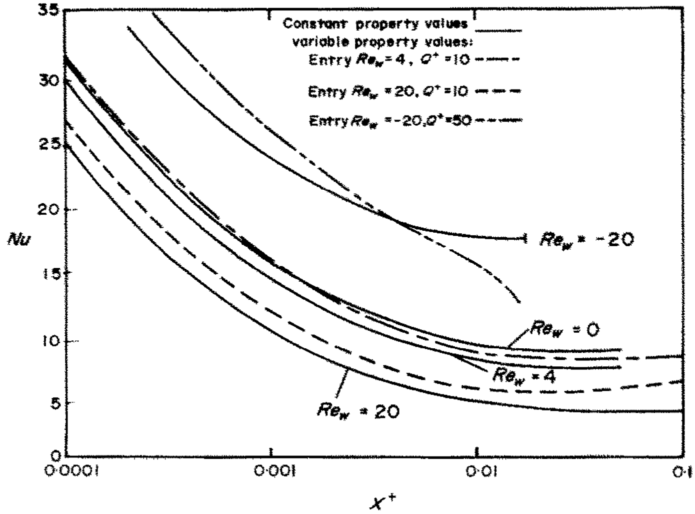

FIG 4. Effect of property variation on Nusselt numbers for parallel porous plates, constant wall heat flux.

thermal entry region over the constant property value for the same entry value of Re_w. Somewhat similarly for flows with suction, the Nusselt number is initially higher than the constant property value but later drops below it due to the increasing wall Reynolds number.

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TRANSFERT THERMIQUE POUR UN ECOULEMENT LAMINAIRE D'UN GAZ A PROPRIETES VARIABLES DANS LA REGION D'ENTREE DE PLAQUES POREUSES PARALLELES

Résumé—L'effet principal de la variation de propriété est d'accroitre le nombre de Nusselt dans la région d'entrée thermique. Des recherches antérieures ont donné des solutions aux problèmes de l'écoulement et du transfert thermique dans le domaine entier des nombres de Reynolds et de Péclet pour un modèle d'écoulement laminaire, incompressible, permanent et à propriété constante. Cet article donne une indication sur les limitations d'un tel modèle. Pour l'injection, le nombre de Mach et celui de Reynolds de écoulement principal en canal dépasseront éventuellement les limites des hypothèses de l'écoulement laminaire incompressible. Pour la succion, les estimations sur le transfert thermique ne sont plus valables près du point d'extraction massique complète puisque la conduction thermique axiale devient alors importante et ne peut être supposée négligeable.

WÄRMEÜBERTRAGUNG IM EINLAUFBEREICH VON PORÖSEN PARALLEN PLATTEN BEI LAMINARER GASSTRÖMUNG MIT VERÄNDERLICHEN STOFFEIGENSCHAFTEN

Zusammenfassung—Der Haupteffekt veränderlicher Stoffeigenschaften führt zu einem leichten Anstieg der Nusselt-Zahl in der thermischen Einlaufzone. Frühere Arbeiten gaben Lösungen für Strömungs- und Wärmeübertragungsprobleme über den ganzen Bereich der Reynolds- und Prandtl-Zahlen an, für ein Modell der laminaren, inkompressiblen, stationären Strömung mit konstanten Stoffeigenschaften. Diese Veröffentlichung zeigt nun die Grenzen dieses Modells. Speziell für Ausblasung kann die Mach-Zahl und die Reynolds-Zahl der Kanalströmung die Grenzen der Voraussetzungen für inkompressible und laminare Strömung übersteigen. Für Absaugung verlieren die Vorausberechnungen für die Wärmeübertragung ihre Gültigkeit in der Nähe der vollständigen Massenstromabsaugung, da dann die axiale Wärmeleitung spürbar wird und nicht mehr vernachlässigt werden kann.

ТЕПЛОПЕРЕНОС ПРИ ЛАМИНАРНОМ ТЕЧЕНИИ ГАЗА С ПЕРЕМЕННЫМИ СВОЙСТВАМИ НА НАЧАЛЬНОМ УЧАСТКЕ КАНАЛА, ОБРАЗОВАННОГО ПАРАЛЛЕЛЬНЫМИ ПОРИСТЫМИ ПЛАСТИНАМИ

Аннотация—Основным эффектом изменения свойств является незначительное увеличение числа Нуссельта в тепловом начальном участке. В результате предыдущих исследований были решены задачи по гидродинамике и теплообмену для всего диапазона значений чисел Рейнольдса и Пекле на стенке в случае стационарного ламинарного течения несжимаемого газа с постоянными свойствами. В работе даются ограничения для такой модели. В частности, при вдуве значения чисел Маха и Рейнольдса для основного потока в канале в конечном итоге будут превышать предельные значения, допустимые для приближения несжимаемого газа и ламинарного течения. При отсосе расчётные данные по теплообмену не могут быть справедливыми вблизи точки полного отвода массы, так как аксиальная теплопроводность в этом случае становится существенной и ею нельзя пренебрегать.